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# Thermodynamic simulation of performance of an Otto cycle with heat transfer and variable specific heats of working fluid

Yanlin Ge<sup>a</sup>, Lingen Chen<sup>a,\*</sup>, Fengrui Sun<sup>a</sup>, Chih Wu<sup>b</sup>

<sup>a</sup> *Faculty 306, Naval University of Engineering, Wuhan 430033, PR China* <sup>b</sup> *Mechanical Engineering Department, U.S. Naval Academy, Annapolis, MD21402, USA*

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#### **Abstract**

The performance of an air-standard Otto cycle with heat transfer loss and variable specific heats of working fluid is analyzed by using finite-time thermodynamics. The relations between the power output and the compression ratio, between the thermal efficiency and the compression ratio, as well as the optimal relation between power output and the efficiency of the cycle are derived by detailed numerical examples. Moreover, the effects of heat transfer loss and variable specific heats of working fluid on the cycle performance are analyzed. The results show that the effects of heat transfer loss and variable specific heats of working fluid on the cycle performance are obvious, and they should be considered in practice cycle analysis. The results obtained in this paper may provide guidance for the design of practice internal combustion engines.

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*Keywords:* Finite-time thermodynamics; Otto cycles; Heat resistance; Performance optimization

### **1. Introduction**

A series of achievements have been made since finitetime thermodynamics was used to analyze and optimize real heat engines [1–3]. Mozurkewich et al. [4] and Hoffman et al. [5] used mathematical techniques from optimal-control theory to determine the optimal motion of the piston in Otto and Diesel cycles. Aizenbud et al. [6] and Chen et al. [7] analyzed the internal combustion engine cycle by using the optimal motion law of a piston fitted to a cylinder containing a gas pumped with a given heating rate. Orlov et al. [8] obtained the power and efficiency limits for internal combustion engines. Angulo-Brown et al. [9], Chen et al. [10] and Wang et al. [11] modeled Otto, Diesel and Dual cycles with friction-like loss during a finite time. Klein [12] studied the effect of heat transfer on the performance of the Otto

and Diesel cycles. Chen et al. [13,14] and Lin et al. [15] derived the relations between net power and the efficiency of the Diesel, Otto and Dual cycles with consideration of heat transfer loss. Chen et al. [16,17] and Ge et al. [18] derived the characteristics of power and efficiency for Otto, Dual and Miller cycles with heat transfer and friction-like term losses. Chen et al. [19], Al-Sarkhi et al. [20], and Sahin et al. [21] studied the optimal power density characteristics for Atkinson, Miller and Dual cycles without any loss. Qin et al. [22] obtained the universal power and efficiency characteristics for irreversible reciprocating heat engine cycles with heat transfer and friction-like term losses. Parlak et al. [23] optimized the performance of irreversible Dual cycle and gave out the experimental results. Fischer et al. [24] found that a quantitative simulation of an Otto engine can be accurately rendered by a simple Novikov model with heat leak. The above work was done without considering the variable specific heats of working fluid, so Ghatak et al. [25] analyzed the effects of the variable specific heat of working fluid and heat transfer loss on the performance of Dual cycle. Based on Ref. [14], this paper will study the effects of the variable

<sup>\*</sup> Corresponding author. Tel.: +86 27 83615046, fax: +86 27 83638709. *E-mail addresses:* lingenchen@hotmail.com, lgchenna@yahoo.com

<sup>(</sup>L. Chen).

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Fig. 1. *T* –*s* diagram for the cycle model.

specific heats of working fluid and heat transfer loss on the performance of Otto cycle.

# **2. Cycle model**

An air standard Otto cycle model is shown in Fig. 1. The compression process is an isentropic process  $1 \rightarrow 2$ ; the heat addition is an isochoric process  $2 \rightarrow 3$ ; the expansion process is an isentropic process  $3 \rightarrow 4$ ; and the heat rejection is an isochoric process  $4 \rightarrow 1$ . As is usual in finite time thermodynamic heat engine cycle models we suppose two instantaneous adiabatic processes  $1 \rightarrow 2$  and  $3 \rightarrow 4$ . For the heat addition and heat rejection branches ( $2 \rightarrow 3$  and  $4 \rightarrow 1$ ) in Fig. 1, we assume that heating from state 2 to state 3 and cooling from state 4 to state 1 proceed according to constant temperature rates, i.e.,

$$
\frac{dT}{dt} = \frac{1}{K_1} \quad (\text{for } 2 \to 3)
$$
  

$$
\frac{dT}{dt} = \frac{1}{K_2} \quad (\text{for } 4 \to 1)
$$
 (1)

where  $T$  is the absolute temperature and  $t$  is time,  $K_1$  and  $K_2$  are constants. Integrating Eq. (1) yields

$$
t_1 = K_1(T_3 - T_2), \qquad t_2 = K_2(T_4 - T_1) \tag{2}
$$

where  $t_1$  and  $t_2$  are the heating and cooling times, respectively. Then, the cycle period is

$$
\tau = t_1 + t_2 = K_1(T_3 - T_2) + K_2(T_4 - T_1) \tag{3}
$$

In practice cycle, specific heats of the working fluid are variable and this variation will have great influence on the performance of the cycle. According to Ref. [25], it can be supposed that the specific heats of the working fluid are functions of temperature alone and over the temperature ranges generally encountered for gases in heat engines (300– 2200 K) the specific heat curve is nearly a straight line which may be closely approximated in the following forms

$$
C_{pm} = a_p + k_1 T \tag{4}
$$

$$
C_{vm} = b_v + k_1 T \tag{5}
$$

where  $a_p$ ,  $b_v$  and  $k_1$  are constants,  $C_{pm}$  and  $C_{vm}$  are molar specific heats with constant pressure and volume, respectively. Accordingly, one has

$$
R = C_{pm} - C_{vm} = a_p - b_v \tag{6}
$$

where  $R$  is the molar gas constant of the working fluid.

The heat added to the working fluid during process  $2 \rightarrow 3$ is

$$
Q_{\text{in}} = M \int_{T_2}^{T_3} C_{vm} dT = M \int_{T_2}^{T_3} (b_v + k_1 T) dT
$$
  
=  $M \left[ b_v (T_3 - T_2) + 0.5 k_1 (T_3^2 - T_2^2) \right]$  (7)

The heat rejected by the working fluid during process  $4 \rightarrow 1$  is

$$
Q_{\text{out}} = M \int_{T_1}^{T_4} C_{vm} dT = M \int_{T_1}^{T_4} (b_v + k_1 T) dT
$$
  
=  $M \left[ b_v (T_4 - T_1) + 0.5 k_1 (T_4^2 - T_1^2) \right]$  (8)

where *M* is the molar number of the working fluid.

Since  $C_{pm}$  and  $C_{vm}$  are dependent on temperature, adiabatic exponent  $k = C_{pm}/C_{vm}$  will vary with temperature. Therefore, the equation often used in reversible adiabatic process with constant *k* cannot be used in reversible adiabatic process with variable *k*. However, according to Ref. [25], a suitable engineering approximation for reversible adiabatic process with variable *k* can be made, i.e., this process can be broken up into infinitesimally small processes, for each of these processes, adiabatic exponent *k* can be regarded as constant. For example, any reversible adiabatic process between states *i* and *j* can be regarded as consisting of numerous infinitesimally small process with constant *k*. For any of these processes, when small changes in temperature  $dT$ , and in volume  $dV$  of the working fluid take place, the equation for reversible adiabatic process with variable *k* can be written as follows

$$
TV^{k-1} = (T + dT)(V + dV)^{k-1}
$$
\n(9)

From Eq. (9), one gets

$$
k_1(T_j - T_i) + b_v \ln(T_j/T_i) = -R \ln(V_j/V_i)
$$
 (10)

The compression ratio is defined as

$$
\gamma = V_1 / V_2 \tag{11}
$$

Therefore, equations for processes  $1 \rightarrow 2$  and  $3 \rightarrow 4$  are as follows

$$
k_1(T_2 - T_1) + b_v \ln(T_2/T_1) = R \ln \gamma \tag{12}
$$

$$
k_1(T_3 - T_4) + b_v \ln(T_3/T_4) = R \ln \gamma \tag{13}
$$

For an ideal Otto cycle model, there are no heat transfer losses. However, for a real Otto cycle, heat transfer irreversibility between working fluid and the cylinder wall is not negligible. We assume that the heat loss through the cylinder wall is proportional to average temperature of both the working fluid and the cylinder wall and that the wall temperature is constant. The heat added to the working fluid by combustion is given in the following linear relation [8,12–14,18,22]

$$
Q_{\rm in} = M \big[ A - B(T_2 + T_3) \big] \tag{14}
$$

where *A* and *B* are two constants related to combustion and heat transfer.

Thus, the power output is

$$
P_{\text{ot}} = \frac{W}{\tau} = \left\{ M \left[ b_v (T_3 + T_1 - T_2 - T_4) + 0.5k_1 (T_3^2 + T_1^2 - T_2^2 - T_4^2) \right] \right\}
$$

$$
\times \left[ K_1 (T_3 - T_2) + K_2 (T_4 - T_1) \right]^{-1} \tag{15}
$$

and the efficiency of the cycle is

$$
\eta_{\text{ot}} = \frac{W}{Q_{\text{in}}} = \left[ b_v (T_3 + T_1 - T_2 - T_4) + 0.5k_1 (T_3^2 + T_1^2 - T_2^2 - T_4^2) \right] \times \left[ b_1 (T_3 - T_2) + 0.5k_1 (T_3^2 - T_2^2) \right]^{-1} \tag{16}
$$

When  $\gamma$  and  $T_1$  are given,  $T_2$  can be obtained from Eq. (12), then, substituting Eq. (7) into Eq. (14) yields  $T_3$ , and the last,  $T_4$  can be worked out by Eq. (13). Substituting  $T_1$ ,  $T_2$ ,  $T_3$  and  $T_4$  into Eqs. (15) and (16) yields the power and efficiency. Then, the relations between the power output and the compression ratio, between the thermal efficiency and the compression ratio, as well as the optimal relation between power output and the efficiency of the cycle can be derived.

#### **3. Numerical examples and discussion**

According to Ref. [25], the following parameters are used:  $A = 60000-70000 \text{ J} \cdot \text{mol}^{-1}$ ,  $B = 20-30 \text{ J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$ ,  $b_v = 19.868 - 23.868$  J·mol<sup>-1</sup>·K<sup>-1</sup>,  $M = 1.57 \times 10^{-5}$  kmol, *<sup>T</sup>*<sup>1</sup> <sup>=</sup> 350 K, *<sup>k</sup>*<sup>1</sup> <sup>=</sup> <sup>0</sup>*.*003844–0*.*009844 J·mol−1·K−2. Taking equal heating and cooling times  $t_1 = t_2 = \tau/2 = 16.6$  ms  $(\tau = 33.33 \text{ ms } [4])$ , the constant temperature rates  $K_1$  and *K*<sub>2</sub> are estimated as  $K_1 = 8.128 \times 10^{-6}$  s·K<sup>-1</sup> and  $K_2 =$  $18.67 \times 10^{-6}$ s·K<sup>-1</sup>.

The variations in the temperatures  $T_2$ ,  $T_3$  and  $T_4$  with the compression ratio are shown in Fig. 2. One can see that *T*<sup>3</sup> and *T*<sup>4</sup> decrease with the increase of compression ratio, and *T*<sub>2</sub> increases with the increase of compression ratio. In Fig. 2, there are two special states: one is the state with  $\gamma = 1$ , and in this case,  $T_4 = T_3$  and  $T_2 = T_1$  hold; the another is the state with  $\gamma = 34.5$ , and in this case,  $T_4 = T_1$  and  $T_2 = T_3$ hold. In the two special states, the power output of the cycle is zero. Figs. 3–6 show the effects of the heat transfer







Fig. 3. The influences of *B* on the power output.

loss on the cycle performance. One can see that the power versus compression ratio characteristic and the power versus efficiency characteristic are parabolic-like curves. For any given *γ* , when the heat transfer loss increases, i.e., *A* decreases or *B* increases, the power output, the working range of the cycle, as well as the efficiency at the maximum power point will become smaller. If *B* increases by about 50%, the maximum power of the cycle decreases by about 28%, and the efficiency at the maximum power point decreases by about 20%. If *A* decreases by about 14%, the maximum power decreases by about 14%, and the efficiency at the maximum power point decreases about 8%.

Figs. 7–12 show the effects of the variable specific heats of the working fluid on the performance of the cycle. Figs. 7– 9 reflects the effects of  $b_v$  on the performance of the cycle. One can see that for any given  $\gamma$ , the power and the working range of the cycle decrease with the decrease of  $b_v$ , while the efficiency increases with the decrease of  $b<sub>v</sub>$ . It also can be found that the decrease of  $b<sub>v</sub>$  almost have no effects on the efficiency at the maximum power point of the cycle. If  $b_v$  decreases by about 17%, the maximum power decreases



Fig. 4. The influences of *B* on the power output versus efficiency characteristic.



Fig. 5. The influences of *A* on the power output.



Fig. 6. The influences of *A* on the power output versus efficiency characteristic.



Fig. 7. The influences of  $b_v$  on the power output.



Fig. 8. The influences of  $b_v$  on the efficiency.



Fig. 9. The influences of  $b_v$  on the power output versus efficiency characteristic.



Fig. 10. The influences of  $k_1$  on the power output.



Fig. 11. The influences of  $k_1$  on the efficiency.

by about 14%. Figs.  $10-12$  show the effects of  $k_1$  on the performance of the cycle. It can be found that the effects of *k*<sup>1</sup> on the performance of the cycle is related to compression ratio  $\gamma$ . If  $\gamma$  is less than certain value, the decrease of  $k_1$  will make the power bigger, on the contrast, if  $\gamma$  exceeds certain value, the decease of  $k_1$  will make the power less. One also can see that the maximum power, and the efficiency at the maximum power point decrease with the decrease of  $k_1$ . The maximum power increases by about 18% and the efficiency at the maximum power point increases by about 10% if *k*<sup>1</sup> increases by about 61%.

In order to observe the practice meaning, one can compare the performance of the Otto cycle with constant molar specific heat and variable molar specific heat. Fig. 13 shows the power output versus compression ratio characteristic with  $k_1 = 0.005844$  J·mol<sup>-1</sup>·K<sup>-2</sup> and  $k_1 = 0$  J·mol<sup>-1</sup>·K<sup>-2</sup>. One can see that for the case of  $k_1 = 0.005844$  J·mol<sup>-1</sup>·K<sup>-2</sup>, the optimum compression ratio at maximum power output point is  $\gamma \approx 11$ . This is consistent with the practical work-



Fig. 12. The influences of  $k_1$  on the power output versus efficiency characteristic.



Fig. 13. The power output versus compression ratio with and without considering variable specific heats of working fluid.

ing compression ratio of SI engines, which are between 9.0 and 11.5 in general.

According to above analysis, it can be found that the effects of the heat transfer losses and the variable specific heat of the working fluid on the cycle performance are obvious, and they should be considered in practice cycle analysis in order to make the cycle model be more close to practice.

# **4. Conclusion**

In this paper, an air standard Otto cycle model with the consideration of the heat transfer and the variable specific heats of working fluid was presented. The performance characteristic of the cycle was obtained by detailed numerical examples. The results show that the effects of the heat transfer loss and variable specific heats of working fluid on the cycle performance are obvious, and they should be considered in practice cycle analysis. The results obtained in this paper may provide guidance for the design of practice internal combustion engines. It would be more meaningful if one considers experimental results. This will be a next work in the near future.

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